Computer Security: Public Key Crypto

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Outline

The blessings of crypto

Using crypto . . .

- Alice can protect her private data
- Alice and Bob can set up a secure channel
  - ensure confidentiality of content
  - ensure authenticity of messages
  - with respect to any adversary Eve
  - over any communication medium
- GlobalCorp. Inc. can protect its business
  - secure financial transactions
  - hide customer database from competitors
  - patch its products in the field for security/functionality
  - protect intellectual property in software, media, etc.
  - enforce its monopoly on games/accessories/etc.

The curse of crypto

- Alice and Bob need to share a secret cryptographic key
- GlobalCorp. Inc. needs to *roll out* many cryptographic keys
- . . . in a way such that Eve cannot get her hands on them
- The security is only as good as the secrecy of these keys

Important lesson:

- Cryptography does not solve problems, but only reduces them to . . .
  - securely generating cryptographic keys
  - securely establishing or rolling out cryptographic keys
  - keeping the keys out of Eve's hands

Key establishment

How do Alice and Bob establish a shared secret?

- When they physically meet:
  - exchange on a piece of paper or business card (unique pairs)
  - on a USB stick: requires trust in stick and PC/smartphone
  - but all cryptography requires trust in devices!
- When they don't meet it is harder. Two cases:
  - there is a common and trusted *friend*: TTP
  - no such friend
- For GlobalCorp. Inc. key management is much harder
  - *Eve* is ubiquitous
  - keys must be protected *in the field*

Remote key establishment w/o trusted third party

- Tamper-evident physically unclonable envelopes
  - tamper-evident: you cannot open it without leaving traces
  - unclonable: cannot fabricate one *looking the same*
- Sending by secure envelope:
  - Alice sticks a 5 Euro banknote on the envelope with superglue
  - Alice writes down the serial number of the banknote
  - Alice sends a key K to Bob in the envelope
  - upon receipt Bob checks that the envelope has not been opened
  - Bob calls Alice and they check the banknote's serial number
  - Bob gets the key K from the envelope

Expensive and time-consuming

Keys management challenges for GlobalCorp. Inc.

Some examples

- Bank: getting keys in all banking cards
- Microsoft: getting software verification key in all PCs
- Spotify or NetFlix: getting keys in user PC/laptop/smartphones
- Government: getting keys in ID cards and travel passports
- More complex eco-systems
  - WWW: establishing keys between User PCs and internet sites
  - Public sector: keys in OV-Chipkaart and readers
  - Mobile phone: ensuring billing and confidentiality while roaming
- etc.

Public Key cryptography to the rescue!

Public key crypto wish list

It would be nice to:

- Authenticate an entity without sharing a key with that entity
- Authenticate documents without writer's secret key:
  - Electronic Signatures!
- Set up a key remotely without the need for secret channel

Public key cryptography can do all that!

. . . and much more

Public key crypto functionality

Public key crypto involves a counter-intuitive idea: use one **key pair** per user, consisting of

- private key PrK: never to be revealed to the outside world
- public key PK: to be published and distributed freely

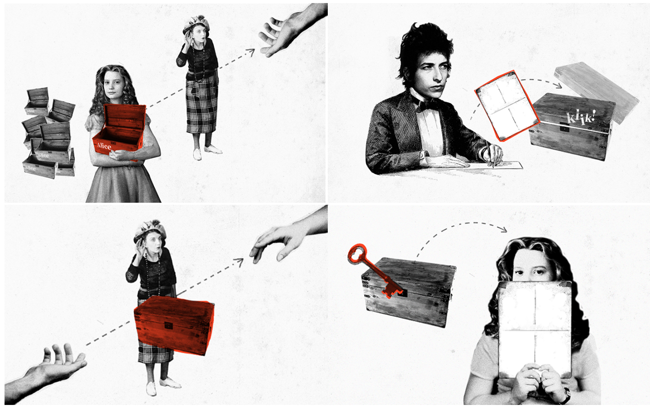
There are different types of public-key cryptosystems. Most used:

- Signature schemes
  - Alice uses PrK_A for signing message: $m, [m]_{PrK_A}$
  - anyone can use PK_A for verifying Alice's signatures
- Encryption schemes
  - using PK_A anyone can encipher a message for Alice $\{m\}_{PK_a}$
  - only Alice can decipher cryptogram with PrK_A
- Key establishment
  - Bob uses PrK_B and PK_A to compute secret K_{AB}
  - Alice uses PrK_A and PK_B to compute secret K_{AB}

Public key encryption as a *form of translation*

- Translation dictionaries
  - Private key PrK is Dictionary Ourgeze to Dutch
  - Public key PK is Dictionary Dutch to Ourgeze
- Say Alice keeps the last copy of the Dictionary Ourgeze to Dutch
  - Encryption: translate to Ourgeze using PK
  - Decryption: translate from Ourgeze using PrK
- Private key PrK can be reconstructed from public key PK!
  - Not secure?
  - In pre-computer time this was a huge task!
- Same for actual public key cryptography
  - PrK can in principle be computed from PK
  - but turns out to be extremely difficult in practice
    - *many tried but none succeeded (so far)*
    - this is the basis of quasi all cryptographic security!

Public key encryption as *self-locking boxes*



Public key crypto: some history

- The idea of public key crypto and first key-establishment scheme
  - Ralph Merkle, Withfield Diffie, Martin Hellman in 1976
  - supposedly already invented at GCHQ in 1969
- The first public key signature and encryption scheme
  - published by Rivest, Shamir and Adleman (RSA) in 1978
  - supposedly already invented at GCHQ in 1970
- Elliptic Curve Cryptography
  - published independently by Koblitz and Miller in 1985
  - GCHQ must have overlooked this
  - the dominant public key cryptosystem today
- Nowadays literally thousands of public key systems

Current trend: post-quantum crypto

- Quantum computer
  - Hypothetical computer that would break all conventional public key crypto
  - Very exotic: *computes in superposition*
  - NSA/GCHQ, Google, IBM, etc. could possibly build one
- Needed: public-key crypto that resists quantum attacks
- European project PQCRYPTO, see http://pqcrypto.eu.org/
- NIST contest for post-quantum crypto, deadline end November
- Active involvement of Radboud colleagues

Some notation that you should know

- $\mathbb{Z}$: the set of integers: $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- $a \in A$: this means that a is an element of a set A. For example, $2 \in \mathbb{Z}$ means 2 is an element of the set of integers, or equivalently, 2 is an integer
- $\forall$: *for all*. E.g., $\forall a \in \mathbb{Z} : a + 1 \in \mathbb{Z}$ means: for every element of the set of integers, that element plus one is also an integer
- $\exists$: *exists*. E.g., $\forall a \in \mathbb{Z}, \exists b \in \mathbb{Z} : a + b = 0$ means: for every integer there exists an integer that added to that integer gives 0
- $|n|$: the length of the integer n in bits

Prime numbers and factorization

- A number is prime if it is divisible only by 1 and by itself.
  Prime numbers are: 2, 3, 5, 7, 11, 13, ... (infinitely many)
- Each number can be written in a unique way as product of primes (possibly multiple times), as in:

$$30 = 2 \cdot 3 \cdot 5 \qquad 100 = 2^2 \cdot 5^2 \qquad 12345 = 3 \cdot 5 \cdot 823$$

- Finding such a prime number factorisation is a computationally hard problem
- In particular, given two very large primes p, q, you can publish $n = p \cdot q$ and no-one will (easily) find out what p, q are.
- Easy for $55 = 5 \cdot 11$ but already hard for $1763 = 41 \cdot 43$
- In 2009 factoring a 232-digit (768 bit) number $n = p \cdot q$ with hundreds of machines took about 2 years

Modular (clock) arithmetic

- On a 12-hour clock, the time '**1 o'clock**' is the same as the time '**13 o'clock**'; one writes

  $1 \equiv 13 \pmod{12}$ ie "1 and 13 are the same modulo 12"

- Similarly for 24-hour clocks:

$$\begin{aligned} 5 &\equiv 29 \pmod{24} && \text{since } 5 + 24 = 29 \\ 5 &\equiv 53 \pmod{24} && \text{since } 5 + (2 \cdot 24) = 53 \\ 19 &\equiv -5 \pmod{24} && \text{since } 19 + (-1 \cdot 24) = -5 \end{aligned}$$

- In general, for $N > 0$ and $n, m \in \mathbb{Z}$,

$$n \equiv m \pmod{N} \iff \text{there is a } k \in \mathbb{Z} \text{ with } n = m + k \cdot N$$

  In words, the difference of n, m is a multiple of N.

Numbers modulo N

How many numbers are there modulo N?

One writes $\mathbb{Z}_N$ for the set of numbers modulo N. Thus:

$$\mathbb{Z}_N = \{0, 1, 2, \cdots N-1\}$$

For every $m \in \mathbb{Z}$ we have $m \bmod N \in \mathbb{Z}_N$.

Some Remarks

- Sometimes $\mathbb{Z}/N\mathbb{Z}$ is written for $\mathbb{Z}_N$
- Formally, the elements m of $\mathbb{Z}_N$ are *equivalence classes* $\{k \mid k \equiv m \pmod N\}$ of numbers modulo N
- These classes are also called residue classes or just residues
- In practice we treat them simply as numbers

Residues form a "ring"

- Numbers can be added (subtracted) and multiplied modulo N: they form a "ring"
- For instance, modulo $N = 15$

$$\begin{array}{rr} 10+6 \equiv 1 & 6-10 \equiv 11 \\ 3+2 \equiv 5 & 0-14 \equiv 1 \\ 4\cdot 5 \equiv 5 & 10\cdot 10 \equiv 10 \end{array}$$

- Sometimes it happens that a product is 1
  For instance (still modulo 15): $4 \cdot 4 \equiv 1$ and $7 \cdot 13 \equiv 1$
- In that case one can say:

$$\frac{1}{4} \equiv 4 \qquad \text{and} \qquad \frac{1}{7} \equiv 13$$

Multiplication tables

For small N it is easy to make multiplication tables for $\mathbb{Z}_N$.

For instance, for $N = 5$,

$\mathbb{Z}_5$	**0**	**1**	**2**	**3**	**4**
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

- **Note**: every non-zero number $n \in \mathbb{Z}_5$ has a an inverse $\frac{1}{n} \in \mathbb{Z}_5$
- This holds for every $\mathbb{Z}_p$ with p a prime number (more below)

Mod and div, and Java (and C too)

- For $N > 0$ and $m \in \mathbb{Z}$ we write $m \bmod N \in \mathbb{Z}_N$
  - $k = (m \bmod N)$ if $0 \leq k < N$ with $k = m + x \cdot N$ for some x
  - For instance $15 \bmod 10 = 5$ and $-6 \bmod 15 = 9$
- % is Java's remainder operation. It behaves differently from mod, on negative numbers.

$$\begin{array}{rr} 7 \,\%\, 4 = 3 & 7 \bmod 4 = 3 \\ -7 \,\%\, 4 = -3 & -7 \bmod 4 = 1 \end{array}$$

  This interpretation of % is chosen for implementation reasons.

  [One also has $7 \,\%\, -4 = 3$ and $-7 \,\%\, -4 = -3$, which are undefined for mod]
- We also use integer division *div*, in such a way that:

$$n = m \cdot (n \; div \; m) + (n \bmod m)$$

  E.g., $15 \; div \; 7 = 2$ and $15 \bmod 7 = 1$, and $15 = 7 \cdot 2 + 1$.

Addition modulo N forms a *group*

The addition satisfies following properties:

closed:	$\forall a, b \in \mathbb{Z}_N$:	$a + b \in \mathbb{Z}_N$
associative:	$\forall a, b, c \in \mathbb{Z}_N$:	$(a + b) + c = a + (b + c)$
neutral element:	$\forall a \in \mathbb{Z}_N$:	$a + 0 = 0 + a = a$
inverse element:	$\forall a \in \mathbb{Z}_N, -a \in \mathbb{Z}_N$:	$a + (-a) = (-a) + a = 0$
abelian (optional)	$\forall a, b \in \mathbb{Z}_N$	$a + b = b + a$

Terminology: Group order

Order of a finite group $(\mathbb{Z}_N, +)$, denoted $\#\mathbb{Z}_N$, is number of elements in the group

Cyclic behaviour in $(\mathbb{Z}_N, +)$

- Consider the sequence (that may cycle):
  - $i = 1 : a$
  - $i = 2 : a + a$
  - $i = 3 : a + a + a$
  - $\ldots$
  - $i = n : na$
- In $(\mathbb{Z}_N, +)$:
  - $\forall a \in \mathbb{Z}_N$ this sequence is periodic
  - period of this sequence is the *order of* a, denoted $\#a$

Terminology: Order of a group element

The order of an element a, denoted $\#a$, is the smallest integer n such that $na = 0$

Cyclic groups and generators

- Let g be some element of $\mathbb{Z}_N$
- Consider the set $\{0g, 1g, 2g, \ldots\}$
- This is a group, called a *cyclic group*, denoted: $\langle g \rangle$
  - Neutral element $0g$
  - Inverse of ig: $(\#g - i)g$
- g is called generator
- Examples in $\mathbb{Z}_{12}$
  - $\langle 3 \rangle = \{3, 6, 9, 0\}$
  - $\langle 5 \rangle = \{5, 10, 3, 8, 1, 6, 11, 4, 9, 2, 7, 0\}$
- $(\mathbb{Z}_n, +)$ itself is a cyclic group
  - generator: $g = 1$
  - $ig = i$

Example on orders: $(\mathbb{Z}_{21}, +)$

- Order of the group $\mathbb{Z}_{21}$: 21
- Order of element 0: 1
- Order of element 1: 21
- Order of element 2: 21
- Order of element 3: 7
- $\ldots$

Shortcut: *find the smallest* i *such that* $i \cdot x$ *is a multiple of* n

Fact: order of an element in $(\mathbb{Z}_n, +)$

$\#x = n/\gcd(n, x)$ with $\gcd(n, x)$: greatest common divisor of x and n

More general:

Lagrange's Theorem (applied to $\mathbb{Z}_N$)

For any element $a \in \mathbb{Z}_N$: $\#a$ divides N

Greatest common divisor

- Definition:
$$\gcd(n,m) = \text{greatest integer } k \text{ that divides both } n \text{ and } m$$
$$= \text{greatest } k \text{ with } n = k \cdot n' \text{ and } m = k \cdot m', \text{ for some } n', m'$$
- Examples:
$\gcd(20,15) = 5 \quad \gcd(78,12) = 6 \quad \gcd(15,8) = 1$
- Properties:
  - $\gcd(n,m) = \gcd(m,n)$
  - $\gcd(n,m) = \gcd(n,-m)$
  - $\gcd(n,0) = n$

Terminology: relative prime (or coprime)

If $\gcd(n,m) = 1$, one calls n, m relative prime or coprime

Euclidean Algorithm

Property (assume $n > m > 0$):

- $\gcd(n,m) = \gcd(m, n \bmod m)$

This can be applied iteratively until one of arguments is 0

Example:
$$\begin{aligned}\gcd(171,111) &= \gcd(111, 171 \bmod 111) = \gcd(111,60)\\ &= \gcd(60, 111 \bmod 60) = \gcd(60,51)\\ &= \gcd(51, 60 \bmod 51) = \gcd(51,9)\\ &= \gcd(9, 51 \bmod 9) = \gcd(9,6)\\ &= \gcd(6, 9 \bmod 6) = \gcd(6,3)\\ &= \gcd(3, 6 \bmod 3) = \gcd(3,0) = 3\end{aligned}$$

Variant allowing negative numbers:
$$\begin{aligned}\gcd(171,111) &= \gcd(111, 171 \bmod 111) = \gcd(111,-51)\\ &= \gcd(51, 111 \bmod 51) = \gcd(51,9)\\ &= \gcd(9, 51 \bmod 9) = \gcd(9,-3)\\ &= \gcd(3, 9 \bmod 3) = \gcd(3,0) = 3\end{aligned}$$

$(\mathbb{Z}_N, \times)$: A group?

- $\times$: Multiplication modulo N
- are group conditions satisfied?
  - closed: yes!
  - associative: yes!
  - neutral element: 1
  - inverse element: no, 0 has no inverse
- Let us exclude 0: so $(\mathbb{Z}_n \setminus \{0\}, \times)$
- Check properties again with multiplication table
- Examples:
  (1) $(\mathbb{Z}_5 \setminus \{0\}, \times)$: OK!
  (2) $(\mathbb{Z}_{21} \setminus \{0\}, \times)$: NOK!

$(\mathbb{Z}_p^*, \times)$ with prime p: a cyclic group!

- If p is a prime, $\mathbb{Z}_p^*$ denotes $\mathbb{Z}_p$ with 0 removed
- Order of the group is $p-1$
- Group turns out to be cyclic

Multiplicative prime groups

$(\mathbb{Z}_p^*, \times)$ is a cyclic group of order $p-1$

Order of an element in $(\mathbb{Z}_p^*, \times)$

- Consider the sequence (that may cycle):
  - $i = 1 : a$
  - $i = 2 : a \times a$
  - $i = 3 : a \times a \times a$
  - $\ldots$
  - $i = n : a^n$
- The operation a^i is called *exponentiation*
- In $(\mathbb{Z}_p^*, \times)$:
  - $\forall a \in \mathbb{Z}_p^*$ this sequence is periodic
  - period is called the (multiplicative) *order of a*, denoted $\#a$

Specifying the inverse of an element in $(\mathbb{Z}_p^*, \times)$

- Lagrange: order of an element divides order of the group $p - 1$
- So for any element, we have $\#x = (p-1)/m$ for some integer m
- So $x^{p-1} = x^{\#x \cdot m} = \left(x^{\#x}\right)^m = 1^m = 1$
- So $x^{-1} = x^{(p-1)} \cdot x^{-1} = x^{(p-1)-1} = x^{p-2}$
- Problem: this costs $p - 3$ multiplications (at first sight ...)

What about $(\mathbb{Z}_N, \times)$ with $N = pq$ and p, q primes

- We remove 0: $\mathbb{Z}_N \setminus \{0\}$
- Inspection of multiplication table reveals some $a \times b = 0$
  - this implies $a \cdot b = k \cdot N$ for some k
  - a cannot be a multiple of N as $a < N$
  - b cannot be a multiple of N as $b < N$
  - a must be multiple of p or of q
  - same for b
  - so a is not coprime to N and b is not coprime to N

Definition of $\mathbb{Z}_N^*$

$\mathbb{Z}_N^*$ is the set of positive integers smaller than N and coprime to N, so with $\gcd(x, N) = 1$

Is $(\mathbb{Z}_N^*, \times)$ a group?

Definition of $\mathbb{Z}_N^*$

$\mathbb{Z}_N^*$ is the set of positive integers smaller than N and coprime to N, so with $\gcd(x, N) = 1$

Note: if N is a prime, $\mathbb{Z}_N^* = \mathbb{Z}_N \setminus \{0\}$

We can check the group properties:

- Closed: if $\gcd(a, N) = 1$ and $\gcd(b, N) = 1$, then $\gcd(ab, N) = 1$
- Associativity follows from associativity of multiplication
- Neutral element: 1
- Does every element have an inverse?

If we can answer this last question positively, we know $(\mathbb{Z}_N^*, \times)$ is a group

Extended Euclidean Algorithm

The extended Euclidean algorithm returns a pair $x, y \in \mathbb{Z}$ with $n \cdot x + m \cdot y = \gcd(n, m)$

Our earlier example for GCD with 171 and 111:

$$\begin{aligned} -51 &= 171 - 2 \cdot 111 \\ 9 &= 111 + 2 \cdot (-51) \\ 3 &= (-51) + 6 \cdot 9 \\ 0 &= (-9) + 3 \cdot 3 \end{aligned}$$

And now by backward substitution:

$$\begin{aligned} 3 &= (-51) + 6 \cdot 9 \text{ (last equation with non-zero lefthand side)} \\ 3 &= (-51) + 6 \cdot (111 + 2 \cdot (-51)) \text{ (substitution of 9)} \\ 3 &= (-51) + 6 \cdot 111 + 12 \cdot (-51) \\ 3 &= 6 \cdot 111 + 13 \cdot (-51) \\ 3 &= 6 \cdot 111 + 13 \cdot (171 - 2 \cdot 111) \text{ (substitution of 51)} \\ 3 &= 6 \cdot 111 + 13 \cdot 171 - 26 \cdot 111 \\ 3 &= 13 \cdot 171 - 20 \cdot 111 \end{aligned}$$

Extended GCD via tables

Compute $egcd(81, 57)$ via the following steps.

n	m	rem	div	$(y, x - y \cdot div)$
81	57	24	1	$(-7, 3 - (-7) \cdot 1) = (-7, 10)$
57	24	9	2	$(3, -1 - 3 \cdot 2) = (3, -7)$
24	9	6	2	$(-1, 1 - (-1) \cdot 2) = (-1, 3)$
9	6	3	1	$(1, 0 - 1 \cdot 1) = (1, -1)$
6	3	0	2	$(0, 1)$
	|| gcd			

Indeed: $-7 \cdot 81 + 10 \cdot 57 = -567 + 570 = 3 =$ gcd

Extended GCD table invariant

Suppose we have reached this stage:

n	m	rem	div	$(y, x - y \cdot div)$
⋮	⋮	⋮	⋮	⋮
a	b			(u, v)
⋮	⋮	⋮	⋮	⋮
	gcd	0		

Then:

$$a \cdot u + b \cdot v = \text{gcd}$$

Check this at every (up-going) step to detect calculation mistakes.

Relative primes lemma

Relative primes Lemma [Important]

m has multiplicative inverse modulo N (i.e., in $\mathbb{Z}_N$) iff $\gcd(m, N) = 1$

Proof ($\Rightarrow$) Extended gcd yields x, y with $m \cdot x + N \cdot y = \gcd(m, N) = 1$. Taking both sides modulo N gives $m \cdot x \bmod N = 1$, or $x = m^{-1}$
($\Leftarrow$) We have $m \cdot x \equiv 1 \bmod N$ so there is an integer y such that $m \cdot x = 1 + N \cdot y$ or equivalently $m \cdot x - N \cdot y = 1$. Now $\gcd(m, N)$ divides both m and N, so it divides $m \cdot x - N \cdot y = 1$. But if $\gcd(m, N)$ divides 1, it must be 1 itself. □

$(\mathbb{Z}_N^*, \times)$ is a group!

- We showed all group properties except that all elements have an inverse
- But the relative primes lemma states that all elements in $\mathbb{Z}_N^*$ have an inverse
- Multiplicative inverse can be computed with extended Euclidean algorithm
  - can be programmed efficiently
- Moreover, it is commutative as ordinary multiplication is commutative

$(\mathbb{Z}_p^*, +, \times)$ is a field [for info only]

Corollary of relative primes lemma

For p a prime, every non-zero $n \in \mathbb{Z}_p$ has an inverse

$(\mathbb{Z}_p, +, \times)$ is a *field*, meaning:

- $(\mathbb{Z}_p, +)$ is a group
- $(\mathbb{Z}_p \setminus \{0\}, \times)$ is a group
- Distributivity:
  - $(a + b) \times c = (a \times c) + (b \times c)$
  - $c \times (a + b) = (c \times a) + (c \times b)$

What is the order of $(\mathbb{Z}_N^*, \times)$?

We now know $(\mathbb{Z}_N^*, \times)$ is a group but don't know its order

Definition: Euler's totient function

Euler's totient function of an integer N, denoted $\phi(N)$, is the number of integers smaller than and coprime to N.

- Clearly $\#(\mathbb{Z}_N^*, \times) = \phi(N)$
- For prime p, all integers 1 to $p - 1$ are coprime to p: $\phi(p) = p - 1$
- For the product of two primes $N = pq$ we have to exclude:
  - 0
  - multiples of p: p, $2p$, ... $(q - 1)p$, so $q - 1$ of them
  - multiples of q: q, $2q$, ... $(p - 1)q$, so $p - 1$ of them
- So
  $\phi(pq) = pq - (p - 1) - (q - 1) - 1 = pq - p - q + 1 = (p - 1)(q - 1)$
- Note: Computing $\phi(N)$ is as hard as factoring N

Number-theoretic theorems [Background info]

Euler's theorem (Lagrange's theorem applied to $(\mathbb{Z}_N^*, \times)$)

If $\gcd(m, N) = 1$, then $m^{\phi(N)} \equiv 1 \bmod N$

PROOF Write $\mathbb{Z}_N^* = \{x_1, x_2, \ldots, x_{\phi(N)}\}$ and form the product: $x = x_1 \cdot x_2 \cdots x_{\phi(N)} \in \mathbb{Z}_N^*$. Form also $y = (m \cdot x_1) \cdots (m \cdot x_{\phi(N)}) \in \mathbb{Z}_N^*$. Thus $y \equiv m^{\phi(N)} \cdot x$. Since m is invertible the factors $m \cdot x_i$ are all different and equal to a unique y_j; thus $x = y$. Hence $m^{\phi(N)} \equiv 1$. □

Fermat's little theorem

If p is prime and m is not a multiple of p then $m^{p-1} \equiv 1 \bmod p$

PROOF Take $N = p$ in Euler's theorem and use that $\phi(p) = p - 1$. □

Used as primality test for p: try out if $m^{p-1} \equiv 1$ for many m.

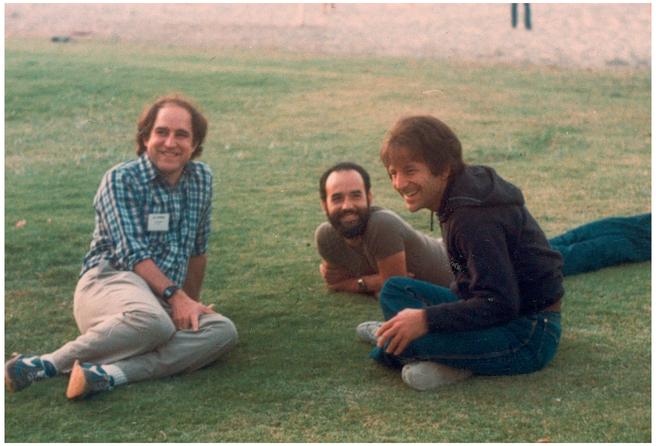
Exponentiation by Square-and-Multiply

- Computing $a^e \bmod n$ in naive way takes $e - 1$ modular multiplications
- Infeasible if a, e and n are hundreds of decimals
- More efficient method: square-and-multiply
- Example: computing g^{12} with *left-to-right* square-and-multiply
  - $g^2 = g \times g$
  - $g^4 = g^2 \times g^2$
  - $g^8 = g^4 \times g^4$
  - $g^{12} = g^8 \times g^4$
- Only 3 squarings and 1 multiplication
- Instead of 11 in naive method

Exponentiation by Square-and-Multiply (cont'd)

- Computing g^{12} with *right-to-left* square-and-multiply
  - $g^2 = g \times g$
  - $g^3 = g^2 \times g$
  - $g^6 = g^3 \times g^3$
  - $g^{12} = g^6 \times g^6$
- Many variants exist, typical computation cost for $a^e \bmod N$:
  - $|e|$ squarings, with $|e|$ the bitlength e
  - 1 to $|e|$ multiplications, depending on e and method
- Relatively cheap
  - This is why group-based public key crypto actually works
  - Computing $x^{-1} \bmod n$ by $x^{\phi(n)-1} \bmod n$ often cheaper than by extended Euclidean algorithm

Ron Rivest, Adi Shamir, Leonard Adleman



Designed their famous cryptosystem in 1977-1978

What is the RSA cryptosystem?

RSA is a *trapdoor one-way function* $y = f(x)$

- given x, computing $y = f(x)$ is easy
- given y, finding x is difficult
- given y and trapdoor info: computing $x = f^{-1}(y)$ is easy

(textbook) encryption with RSA:

$$\{m\}_{PK} = m^e \bmod n$$

(textbook) decryption with RSA:

$$[c]_{PrK} = c^d \bmod n$$

- Public key: $PK = (n, e)$
- Private key: $PrK = (n, d)$
- Modulus $n = p \cdot q$ with p and q large primes
  - the factorization $n = p \cdot q$ is the trapdoor

How to determine the RSA private key

The order of the group $(\mathbb{Z}_n^*, \times)$ is $\phi(n) = (p-1)(q-1)$ so $\forall x \in \mathbb{Z}_n^*$:

$$x^{\phi(n)} \bmod n = x^{(p-1)(q-1)} \bmod n = 1$$

Let d satisfy

$$e \cdot d = 1 + k \cdot (p-1)(q-1)$$

then (omitting $\bmod n$)

$$(x^e)^d = x^{e \cdot d} = x^{1+k\cdot\phi(n)} = x \cdot x^{k\phi(n)} = x \cdot (x^{\phi(n)})^k = x$$

(Conclusion actually holds for all $x \in \mathbb{Z}_n$)

So the RSA private exponent d is given by

$$d = e^{-1} \bmod (p-1)(q-1)$$

Recap: RSA public key pair

- Public key: public exponent and modulus (e, n)
- Private key: private exponent and modulus (d, n)
- Modulus:
  - $n = p \cdot q$ with p and q large primes
- Public exponent e
  - often small prime, e.g., $2^{16} + 1$: makes computing x^e light
  - $p - 1$ and $q - 1$ shall be coprime to e
- Private exponent d
  - exponent d is inverse of e modulo $(p-1)(q-1)$
  - length of d is close to that of n: x^d much slower than x^e
- Security of RSA relies on difficulty of factoring n
  - factoring n allows computing d from (e, n)
  - p and q shall be large enough and unpredictable by attacker
  - given n, knowledge of $\phi(n)$ allows factoring n and computing d

Factoring n, given $\phi(n)$ [for info only]

Assume modulus n is known

Knowledge of $\phi(n)$ allows factoring n

We have two equations $n = p \cdot q$ and $\phi = (p-1)(q-1)$ or

$$n = p \cdot q \text{ and } \phi = p \cdot q - p - q + 1$$

Subtracting them: $\phi = n - p - q + 1$. Reordering and substitution:

$$n = p \cdot (n - \phi + 1 - p)$$

Working out gives the following quadratic equation in p:

$$p^2 - A \cdot p + n = 0 \text{ with } A = n - \phi + 1$$

Using the standard formula for the solutions of a quadratic equation:

$$p, q = \frac{A \pm \sqrt{A^2 - 4n}}{2} \text{ with } A = n - \phi + 1$$

Factoring n, given $\phi(n)$, example [for info only]

So we have:

$$p, q = \frac{A \pm \sqrt{A^2 - 4n}}{2} \text{ with } A = n - \phi + 1$$

Example: $n = 2021$ and $\phi(n) = 1932$.

This yields $A = 2021 - 1932 + 1 = 90$

$$p = \frac{90 + \sqrt{8100 - 4 \cdot 2021}}{2} \text{ and } q = \frac{90 - \sqrt{8100 - 4 \cdot 2021}}{2}$$

So $p = \frac{90+\sqrt{16}}{2} = 47$ and $p = \frac{90-\sqrt{16}}{2} = 43$

Difficulty of factoring

- State of the art of factoring: two important aspects
  - reduction of computing cost: Moore's Law
  - improvements in factoring algorithms
- Factoring algorithms
  - Sophisticated algorithms involving many subtleties
  - Two phases:
    - distributed phase: equation harvesting
    - centralized phase: equation solving
  - Best known: general number field sieve (GNFS)
- These advances lead to increase of advised RSA modulus lengths see http://www.keylength.com/

Factoring records

number	digits	date	sieving time	alg.
C116	116	mid 1990	275 MIPS years	mpqs
RSA-120	120	June, 1993	830 MIPS years	mpqs
RSA-129	129	April, 1994	5000 MIPS years	mpqs
RSA-130	130	April, 1996	1000 MIPS years	gnfs
RSA-140	140	Feb., 1999	2000 MIPS years	gnfs
RSA-155	155	Aug., 1999	8000 MIPS years	gnfs
C158	158	Jan., 2002	3.4 Pentium 1GHz CPU years	gnfs
RSA-160	160	March, 2003	2.7 Pentium 1GHz CPU years	gnfs
RSA-576	174	Dec., 2003	13.2 Pentium 1GHz CPU years	gnfs
C176	176	May, 2005	48.6 Pentium 1GHz CPU years	gnfs
RSA-200	200	May, 2005	121 Pentium 1GHz CPU years	gnfs
RSA-768	232	Dec., 2009	2000 AMD Opteron 2.2 Ghz CPU years	gnfs

Using RSA for encryption

The naive way, called *textbook RSA*:

- Bob enciphers message for Alice with her public key: $c = \{m\}_{PK_A}$
  - codes his message as an integer $m \in \mathbb{Z}$
  - computes $c = m^e \bmod n$, so with $PK_A = (e, n)$
- Alice deciphers received cryptogram with her private key: $m = [c]_{PrK_A}$
  - computes $m = c^d \bmod n$ with $(d, n) = PrK_A$, her private key
  - decodes m as a message

Using RSA for encryption: attention points

Plaintext m shall have enough entropy:

- Otherwise, Eve can guess m and check if $c = m^e \bmod n$

Example: PIN encryption in EMV (Visa, Mastercard) payment cards

- Requirement: protecting PIN against wiretapping of card contacts
- Solution: encryption between terminal and smart card using RSA
- Confidentiality: terminal adds random string r: $m = PIN \| r$
  - Note: in symmetric encryption plaintext uniqueness (nonce) is sufficient
- Freshness: include challenge N from card $m = PIN \| r \| N$

Using RSA for encryption: attention points (cont'd)

Algebraic properties of RSA: (malleability)

- Say Eve has the plaintexts m_1 and m_2 of two cryptograms c_1 and c_2.
- So $m_1 = c_1^d$ and $m_2 = c_2^d$ with $(d, n) = PrK_A$
- Then if she sees a cryptogram that happens to be $c_3 = c_1 \times c_2$, she can decipher it without PrK_A
- Namely: $c_3^d = (c_1 \times c_2)^d = c_1^d \times c_2^d = m_1 \times m_2$
- So Eve can decipher c_3 without known the private key!
- Same for e.g. $c_4 = c_1 \times c_1$, or in general $c_i = c_1^t \times c_2^v$

Other inconvenient properties:

- Length of message m is limited by $|m| \leq |n|$
- RSA decryption is relatively slow

Current advice by experts: don't encipher data with RSA

Using RSA for encryption: solutions

- Apply a hybrid scheme:
  - use RSA for establishing a symmetric key
  - encipher and authenticate with symmetric cryptography
- Sending an encrypted key
  - addition of redundancy and randomness before encryption
  - verification of redundancy after decryption
  - if NOK, return error
- Many proposals:
  - best known standard: PKCS #1 v1.5 and v2 (e.g. OAEP)
  - rather complex and not clear if objectives are achieved
- despite the problems, this is still the most widespread method

Example: PKCS#1 v1.5 padding for encryption

INPUT: Recipient's RSA public key, (n, e) of length $k = |n|/8$ bytes; payload D (e.g., a symmetric key) $|D| \leq 8(k - 11)$.
OUTPUT: Encrypted block of length k bytes

(1) Form the k-byte encoded block, EB

$$EB = 00 \,\|\, 02 \,\|\, PS \,\|\, 00 \,\|\, D$$

where PS is a random string $k - |D| - 3$ non-zero bytes (ie. at least eight random bytes)

(2) Convert byte string EB to integer m.

(3) Encrypt with RSA: $c = m^e \bmod n$

(4) Convert c to k-byte output block OB

(5) Output OB

PKCS#1 v1.5 encryption padding example

Assume a RSA public key (n, e) with n 1024 bit long.

As data D, take a (random) AES-128 key, such as:

$D =$ 4E636AF98E40F3ADCFCCB698F4E80B9F

Message block EB with random padding bytes shown in green:

```
EB = 0002257F48FD1F1793B7E5E02306F2D3
     228F5C95ADF5F31566729F132AA12009
     E3FC9B2B475CD6944EF191E3F59545E6
     71E474B555799FE3756099F044964038
     B16B2148E9A2F9C6F44BB5C52E3C6C80
     61CF694145FAFDB24402AD1819EACEDF
     4A36C6E4D2CD8FC1D62E5A1268F49600
     4E636AF98E40F3ADCFCCB698F4E80B9F
```

The random padding makes $m^e \bmod n$ different each time

Using RSA for encryption: state-of-the-art

RSA Key Establishment Method (KEM)

- Bob randomly generates $r \in \mathbb{Z}_n$
- Bob sends $c = r^e \bmod n$ to Alice
- Alice deciphers c back to r
- both compute shared symmetric key K as $K = \text{hash}(r)$

RSA-KEM is the sound way to use RSA for establishing a key

Using RSA for signatures

The naive way:

- Alice signs message m with her private key PrK_A: $s = [m]_{PrK_A}$
  - codes her message as an integer m in $\mathbb{Z}_n$
  - computes $s = m^d \bmod n$, so with $PrK_A = (d, n)$:

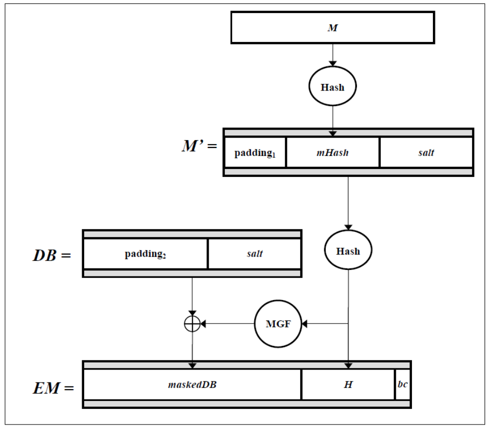
$$s = [m]_{PrK_A} = m^d \bmod n$$

- Bob verifies the signed message (m, s):
  (1) computes $m' = s^e \bmod n$, so with $PK_A = (e, n)$
  (2) checks that $m' = m$

Using RSA for signatures: attention points

- Limitation on message length (and secure modes are hard to define)
  - instead of m, we input $h(m)$
  - additional benefit: becomes much faster
- RSA malleability
  - given two signatures $s_1 = m_1^d$ and $s_2 = m_2^d$, Eve can construct a signature for $m_3 = m_1 \cdot m_2 \bmod n$ by computing $s_3 = s_1 \cdot s_2 \bmod n$.
  - this is forgery: signing without knowing private key
- solution: specific padding schemes, e.g. PKCS # 1 v1.5 or v2 (PSS)
  - adds redundancy by padding
  - applies hashing for destroying malleability
  - e.g., $s_1 \cdot s_2$ no longer verifies as a valid signature

RSA Probabilistic Signature Scheme (PSS) [for info only]



(MGF = XOF)

RSA efficiency

- Private exponentiation:
  - Square and multiply
  - grows with the third power of the modulus length
  - e.g., modulus length $\times 2$: computation time goes $\times 8$
- Public exponentiation:
  - more efficient thanks to short public exponent
- Key generation:
  - randomly generating large primes p and q
  - About 15 to 40 times the effort of a private exponentiation

RSA toy example, by hand [required skill]

Key generation:

- Choose $e = 3$
- Take $p = 5, q = 11$, so that $n = p \cdot q = 55$ and $\phi(n) = 40$
  - OK: both $p - 1$ and $q - 1$ are coprime to e
- Compute $d = \frac{1}{e} = \frac{1}{3} \in \mathbb{Z}^*_{40}$ with extended Euclidean algorithm:
  - it yields $x, y \in \mathbb{Z}$ with $40x + 3y = 1$, so that $d = \frac{1}{3} = y$
  - By hand: $3^{-1} \bmod 40 = -13 = 27$
    (indeed with $40 \cdot 1 + 3 \cdot -13 = 40 - 39 = 1$)

Encryption and decryption of message $m = 19 \in \mathbb{Z}_n$

- encipher: $c = m^e \bmod n = 19^3 \bmod 55 = 39$
- decipher: $m' = c^d \bmod n = 39^{27} \bmod 55 = 19$

Recap: RSA key pair generation

A user generating an RSA key pair with given modulus length $|n|$:

- chooses the public exponent e
  - often a small prime imposed by the context
  - sometimes randomly generated per user, e.g. 256 bits
- randomly generates prime p of given length $\ell = |n|/2$
  - $p - 1$ shall be coprime to e
- randomly generates prime q such that $p \cdot q$ has length $|n|$
  - $q - 1$ shall be coprime to e
- computes modulus $n = p \cdot q$
- computes private exponent d as e^{-1} modulo $(p-1)(q-1)$
- Attention points [for info only]:
  - RSA works with p, q of any length but often software requires that $|n|$ is a multiple of 8 (or 32) and $|p| = |q| = |n|/2$
  - There are multiple valid values of $d < (p-1)(q-1)$ but just one $< \mathrm{lcm}(p-1)(q-1) = (p-1)(q-1)/\gcd(p-1, q-1)$

RSA toy example, calculated by hand [required skill]

- Choose $e = 3$
- Take $p = 5, q = 11$, so that $n = p \cdot q = 55$ and $\phi(n) = 40$
  - OK: both $p - 1$ and $q - 1$ are coprime to e
- Compute $d = \frac{1}{e} = \frac{1}{3} \in \mathbb{Z}^*_{40}$ with extended Euclidean algorithm:
  - it yields $x, y \in \mathbb{Z}$ with $40x + 3y = 1$, so that $d = \frac{1}{3} = y$
  - By hand: $3^{-1} \bmod 40 = -13 = 27$
    (indeed with $40 \cdot 1 + 3 \cdot -13 = 40 - 39 = 1$)
- Let message $m = 19 \in \mathbb{Z}_n$
  - encipher $c = m^e \bmod n = 19^3 \bmod 55 = 39$
  - decipher $m' = c^d \bmod n = 39^{27} \bmod 55 = 19$

The Achilles' Heel of (public key) cryptography

Cryptography does not solve problems, but only reduces them

- In public key cryptography, problems are reduced to:

  Authentication of public keys

- How do we know whether PK_A actually belongs to Alice, when
  - we verify a signature with PK_A?
  - we establish a shared secret using PK_A?
  - we authenticate someone using PK_A?
- PK_A could actually be the public key of Trudy
- Need: authenticate link between public key and its owner
- In many practical systems this issue is not well addressed
  - one of reasons for the miserable level of security in IT
  - same mistakes made again and again (see next slides)
  - problem of human behaviour rather than technology

Methods of public key authentication

Say, Bob wants to use Alice's public key

- He can obtain it via email, Alice's homepage, business card, ...

There are essentially three methods:

- Manual: Bob relies on Alice alone
- Web of trust: Bob relies on their mutual friends
- Certificate Authority (CA): Bob relies on a central authority

... and: Trust on First Use (TOFU): Bob knocks on wood

Systems for public key authentication (and revocation) are called Public Key Infrastructures (PKI). Most of the time, the term PKI is used as a synonym of the CA method.

Manual public key validation

- Bob checks with Alice if his copy of PK_A matches that of Alice
  - e.g., face-to-face, via phone or video-call
  - email will NOT do
  - requires that Bob verifies he is actually talking to Alice
- Often one uses a hash
  - verifying $h(PK_B, Id_B)$ instead of key PK_B directly
  - hash function shall be 2nd preimage resistant
  - reader-friendly coding of the hash: fingerprint
- Most reliable method
  - very rarely used
  - main problem: requires users to be security-aware
- a public key crypto pioneer: Phil Zimmerman
  - 1991: creates PGP secure email, supporting key validation
  - now: at Silent Circle (e.g. blackphone), settling with TOFU
  - *you cannot be idealistic all your life*

Web-of-trust public key authentication

Crowd style ("trust what your friends say", bottom-up)

- Say Alice and Bob have a common friend: Wally
  - Bob already has an authentic copy of PK_W: Wally's public key
  - Wally already verified that his copy of PK_A is authentic
  - Bob asks Wally to sign $\langle \text{Alice}, PK_A \rangle$ with his private key PrK_W
  - Bob can now verify this signature (certificate) using PK_W
- For more assurance, Bob can ask multiple friends to sign $\langle \text{Alice}, PK_A \rangle$
- Wally acts as a kind of TTP
- Difference with the TTP in the symmetric-key case
  - symmetric: TTP has shared key and can cheat undetectedly
  - here Wally can sign $\langle \text{Alice}, PK_{W'} \rangle$ instead of $\langle \text{Alice}, PK_A \rangle$
  - ... and can decipher Bob's messages and/or sign as Alice
  - but: Bob and Alice can catch Wally by manual validation
- Feature introduced by Phil Zimmerman in PGP
  - same problem: requires security-aware users
  - PGP (and gpg) usage in practice nowadays: mostly TOFU

Web of trust: signing parties

- People meet to check each other's identity
- and exchange public key fingerprints: (truncated) hashes of public keys (BJ's is `0xA45AFFF8`)
  - beware of 2nd preimages, so don't truncate too much!
- to later look up the keys corresponding to the fingerprint and sign them

(source: http://xkcd.com/364/)

Certificate Authority

Phone-book style ("trust what an authority says", top-down)

- use a trusted list of pairs $\langle name, PK_{\text{name}} \rangle$
- but who can be trusted to compile and maintain such a list?
- this is done by a Certificate Authority (CA)
  - a *super-Wally* that signs public keys to be trusted by everyone
- Basic notion: public key certificate, i.e. signed statement:

$$\left[\text{``}\textit{Trustee} \text{ declares that the public key of } X \text{ is } PK_X; \\ \text{this statement dates from (}\textit{start date}\text{) and is valid} \\ \text{until (}\textit{end date}\text{), and is recorded with (}\textit{serial nr.}\text{)''}\right]_{PrK_{\textit{Trustee}}}$$

- There are standardised formats for certificates, like X.509
- The term (public key) certificate is often abused

Public Key Infrastructure (PKI)

Two relevant authorities:

- Certification Authority (CA)
  - generates public key certificates
  - publishes certificate revocation lists for compromised keys
  - can be done in multiple levels: root CA and intermediate ones
- Registration Authority
  - part of CA that verifies the identity of the user
  - expensive part, with many administrative and legal aspects

Practically,

- Most CAs are commercial companies, like VeriSign, Thawte, Comodo, or DigiNotar (now "dead")
- They offer different levels of certificates, depending on the thoroughness of identity verification in registration

Example verification, by VeriSign

VeriSign offers three assurance levels for certificates

(1) Class 1 certificate: only email verification for individuals: "authentication procedures are based on assurances that the Subscriber's distinguished name is unique within the domain of a particular CA and that a certain e-mail address is associated with a public key"

(2) Class 2 certificate: "verification of information submitted by the Certificate Applicant against identity proofing sources"

(3) Class 3 certificate: "assurances of the identity of the Subscriber based on the personal (physical) presence of the Subscriber to confirm his or her identity using, at a minimum, a well-recognized form of government-issued identification and one other identification credential."

Where do I find someone else's certificate?

- The most obvious way to obtain a certificate is: directly from the owner
- From a certificate directory or key server, such as:
  - `pgp.mit.edu`
    (you can look up BJ's key there, and see who signed it)
  - `subkeys.pgp.net` *etc.*
- The root public keys are pre-configured, typically in browsers.
  - Often called "root certificates", but they aren't
  - E.g., in *firefox* look under Preferences - Advanced - View Certificates
  - On the web: `www.mozilla.org/projects/security/certs/included`

Certificate (PKI) usage examples

- "Secure webaccess" via server-side certificates, recognisable via:

  - protocols: TLS and https
  - allows user to authenticate website content
  - protects confidentiality of web traffic between user and site
  - important for passwords and card nr. based credit card payments
- Code signing, for integrity and authenticity of downloaded code
- EMV payment with smart cards: VISA, Mastercard, Maestro
- Client-side certificates for secure remote logic (e.g., in VPN = Virtual Private Network)
- National ID cards and travel passports
- Sensor-certificates in a sensor network, against spoofing sensors and/or sensor data

Certificate Revocation, via CRLs

Revocation is: declaring a public key certificate no longer valid

Possible reasons for revocation

- certificate owner lost control over the private key
- crypto has become weak (think of MD5 or SHA-1 hash)
- CA turns out to unreliable (think of DigiNotar)

Certificate Revocation Lists (CRLs)

- maintained by CAs, and updated regularly (e.g., 24 hours)
- should be consulted before every use of a certificate
- you can subscribe to revocation lists so that they are loaded automatically into your browser

Revocation, via OCSP

- In off-line checking, CRLs require bandwidth and local storage
  - overflowing the list is possible attack scenario
- Alternative: OCSP = Online Certificate Status Protocol
  (1) Suppose Bob wants to check Alice's certificate before use
  (2) Bob sends OCSP request to CA with certificate serial nr.
  (3) CA looks up serial number in its (supposedly) secure database
  (4) if not revoked, it replies with a signed, successful OCSP response
- **Privacy issue**: with OCSP you reveal to CA which certificates you use, and thus who you communicate with
  - also when you communicate with someone using OCSP

Note: you are basically online with the CA, so long-term certificates are not really needed.

Certificate chains

Imagine you have certificates:

(1) ["A's public key is PK_A ..."]$_{PrK_B}$

(2) ["B's public key is PK_B ..."]$_{PrK_c}$

Suppose you have these 2 certificates, and C's public key

- What can you deduce?
- Who do you (have to) trust?
- To do what?

Example: active authentication in e-passport

- private key securely embedded in passport chip
- public key signed by producer (*Morpho* in NL)
- Morpho's public key signed by Dutch state

The trouble with PKI

- All participants need authentic copies of root CA public keys
  - a root CA cannot have a certificate, per definition
  - often does have a meaningless *self-signed certificate*
  - hardcoded in software or included in software releases
  - you are trusting Microsoft, Mozilla, Google, Apple, KPN ...
- Why most PKI's have failed up to now:
  - CAs in theory: trustworthy service providers that accept liability
  - CAs in practice: unreliable organizations only in it for the money
- Tension between (CA) PKI concept and the essence of public key crypto:
  - PK crypto: authentication and confidentiality without need for pre-shared keys or trusted third party
  - CA is nothing more than a trusted third party

Problems in the TLS (https) PKI

- In your browser there are about 650 CA root keys
  - Note: a common misnomer for CA root key is (CA) root certificate
  - whatever these CAs sign is shown as trusted by your browser
- This makes the PKI system fragile
  - CAs can sign anything, not only for their customers
  - e.g. rogue gmail certificates, signed by DigiNotar, appeared in aug.'11, but Google was never a customer of DigiNotar
- Available controls are rather weak:
  - rogue certificates can be revoked (blacklisted), after the fact
  - browser producers can remove root certificates (of bad CAs)
  - compulsory auditing of CAs
  - via OCSP server logs certificate usage can be tracked
- root of the problem: lack of liability of software providers and CAs

Free/community CA services

- **CAcert**, `https://cacert.org`
  - provides free certificates, via a web-of-trust
  - certificate owners can accumulate points by being signed by assurers
  - if you have ≥ 100 points, you can become assurer yourself
  - CAcert never managed get its root key into major browsers
- **Let's encrypt**, `https://letsencrypt.org/`
  - more recent initiative for free TLS certificates
  - issued via an automated process, with short (90 day) validity
  - no own root key in browsers, but "cross-signed" version by existing CA (IdenTrust)

In both cases, no liability is accepted.

Trust on first use (TOFU)

Per default, no public key validation

- Bob trusts that received public key is Alice's without validation
- Man-in-the-middle risk: Eve can substitute public key by hers
- Used by the cool crowd:
  - messaging service Signal
  - messaging service Whatsapp
  - secure mobile blackphone from Silent Circle
  - ...
- Sometimes presented as alternative to PKI
- How is it possible that people buy this nonsense?
  - it promises security without the effort, a.o., key management
  - similar to voting for populists and expecting improvement
  - or eating chocolate to feel better
- It is not all bad: systems do support manual key validation

Example of TOFU: WhatsApp

- There is a white paper describing the security protocol
  - not enough detail to know what they are doing exactly
  - e.g. what happens when replacing phone?
  - complex protocol with 4 layers of ECC and 3 of symmetric crypto
- Uses ECC public key pairs to establish symmetric keys
  - public key pairs generated at install time
  - distributed via central WhatsApp server without validation
- Manual validation by select *contact*, item *encryption*
  - not transparent nor user-friendly
- Preliminary conclusion
  - a critical review would be welcome

Note: Whatsapp protocol is based on protocol of Signal, that in turn is open source

DigiNotar I: background

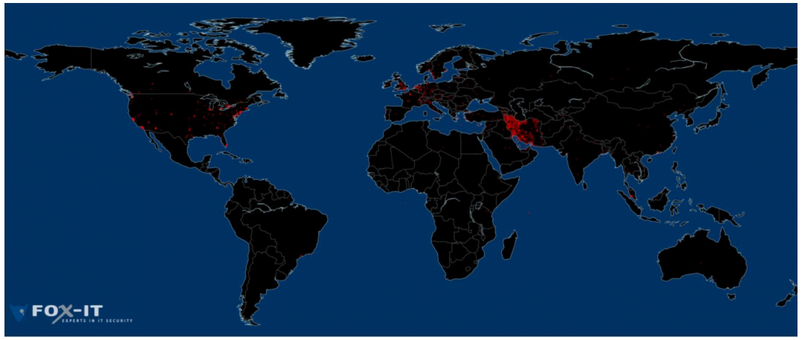
- The Dutch CA DigiNotar was founded in 1997, based on need for certificates among notaries
  - bought by US company *VASCO* in jan'11
  - "voluntary" bankruptcy in sept.'11
- DigiNotar's computer systems were infiltrated in mid july'11, resulting in rogue certificates
  - *DotNetNuke* CMS software was 30 updates ($\geq$ 3 years) behind
  - Dutch government only became aware on 2 sept.
  - it operated in "crisis mode" for 10 days
- About 60.000 DigiNotar certificates used in NL
  - many of them deeply embedded in infrastructure (for inter-system communication)
  - some of them need frequent re-issuance (short-life time)
  - national stand-still was possible nightmare scenario

DigiNotar II: act of war against NL?

- Hack claimed by 21 year old Iranian "Comodohacker"
  - he published proof (correct sysadmin password 'Pr0d@dm1n')
  - claimed to have access to more CAs (including GlobalSign)
  - also political motivation (see `pastebin.com/85WV10EL`)

    Dutch government is paying what they did 16 years ago about Srebrenica, you don't have any more e-Government huh? You turned to age of papers and photocopy machines and hand signatures and seals? Oh, sorry! But have you ever thought about Srebrenica? 8000 for 30? Unforgivable... Never!
- Hacker could have put all 60K NL-certificates on the blacklist
  - this would have crippled the country
  - interesting question: would this be an act of war?
  - difficult but very hot legal topic: attribution is problematic
  - traditionally, in an "act of war" it is clear who did it.

DigiNotar III: rogue certificate usage (via OCSP calls)



Main target: 300K gmail users in Iran (via man-in-the-middle)

(More info: search for: *Black Tulip Update*, or for: *onderzoeksraad Diginotarincident*)

DigiNotar IV: certificates at stake

- DigiNotar as CA had its own root key in all browsers
  - after the compromise, it was kicked out, in browser updates
  - Microsoft postponed its patch for a week (for NL only)!
  - the Dutch government requested this, in order to buy more time for replacing certificates (from other CAs)
- DigiNotar was also sub-CA of the Dutch state
  - private key of *Staat der Nederlanden* stored elsewhere
  - big fear during the crisis: this root would also be lost
  - it did not happen
  - alternative sub-CA's: Getronics PinkRoccade (part of KPN), QuoVadis, DigiDentity, ESG

DigiNotar V: Fox-IT findings

- DigiNotar hired security company *Fox-IT* (Delft)
  - Fox-IT investigated the security breach
  - published findings, in two successive reports (2011 & 2012)
- **Actual problem**: the serial number of a DigiNotar certificate found in the wild was not found in DigiNotar's systems records
- The number of rogue certificates is unknown
  - but OCSP logs report on actual use of such certificates
- Fox-IT reported "hacker activities with administrative rights"
  - attacker left signature *Janam Fadaye Rahbar*
  - same as used in earlier attacks on Comodo
- Embarrassing findings:
  - all CA servers in one Windows domain (no compartimentalisation)
  - no antivirus protection present; late/no updates
  - some of the malware used could have been detected

DigiNotar VI: lessons if you still believe in CA's

- Know your own systems and your vulnerabilities!
- Use multiple certificates for crucial connections
- Strengthen audit requirements and process
  - only management audit was required, no security audit
  - the requirements are about 5 years old, not defined with "state actor" as opponent
- Security companies are targets, to be used as stepping stones
  - e.g., march'11 attack on authentication tokens of RSA company
  - used later in attacks on US defence industry
- Alternative needed for PKI?
- Cyber security is now firmly on the (political) agenda
  - also because of "Lektober" and stream of (website) vulnerabilities
  - now almost weekly topic in Parliament
    (e.g., breach notification and privacy-by-design)

DigiNotar VII: Finally (source: NRC 7/9/2011)

DigiNotar has not re-emerged: it had only one chance and blew it!

Entity authentication with electronic signatures

Challenge-response with electronic signature $[\ldots]_{PrK}$

$$A \longrightarrow B\colon N, Id_A$$
$$B \longrightarrow A\colon [N, Id_A]_{PrK_B}$$

or mutual authentication

$$A \longrightarrow B\colon N_B, Id_A$$
$$B \longrightarrow A\colon [N_B, Id_A]_{PrK_B}, N_A, Id_B$$
$$A \longrightarrow B\colon [N_A, Id_B]_{PrK_A}$$

- Advantage: verifier does not require secret!
  - Prover does not need to trust verifier for protecting its keys
  - Same private key can be used to authenticate in several places
  - This creates privacy issues: linkability

The claim (or myth) of non-repudiation

- The unique advantage of asymmetric crypto is:
  - verification of public key signature does not require a secret key
  - so only the signer could have generated the signature
- Public-key advocates have used this to promote their crypto:

Public-key signatures support non-repudiation

Non-repudiation: inability after signing something to deny it

- A legal/business property is attributed to a cryptographic protocol
- **But** there are excuses for denying a signature, such as:
  - someone else used the private key on my PC or smart card
  - I did sign but not the document you are showing me
  - the crypto has been broken
  - ...
- In the end it is about rules, terms and conditions and agreements

Electronic vs. ordinary signatures

- **Ordinary 'wet' signature**
  - produced by human, expressing clear intent
  - the same on all documents
  - one person typically has one signature
  - easy to forge, but embedded in established usage context
- **Electronic signature**
  - different for each signed document
  - person may have multiple key pairs, e.g., 1 business, 1 personal
  - electronic signatures can be legally recognized
    - EU directive 1999/93/EC, replaced by eIDAS in 2014
    - requires certified secure signature-creation device
    - in practice: an ID chip card containing private key(s)
    - legal validity implies PKI with government-approved CA
    - conditions for NL at `pkioverheid.nl`
  - crypto is mature, deployment still problematic

Electronic signatures, the ID chip card

- The private keys should at all time be under control of the user
  - the ID card signs a string presented to it with its private key(s)
  - requires prior submission of a PIN
  - retrieving the private key from the chip should be hard
  - key pairs should be generated on-card
- Two main use cases:
  - authentication with challenge-response: for access to web sites, infrastructure, etc.
  - document signing, where a hash is presented to a card

  A user should be in control of whether he does one or the other

Electronic signatures, the ID chip card (cont'd)

- Two key pairs:
  - one for *authentication*
  - one for *non-repudiation* (signatures)
- each key has its own PIN
  - so the user is in principle aware of what (s)he is doing
- a more cost-effective solution:
  - a single key pair for both operations
  - two separate PINs for the functions
  - distinguish hashes (sign) from challenges (auth) with domain separation
- Scenario upon presentation of x to chip (single-key case)
  - x can be $h(m)$ or a challenge
  - if *sign* PIN was presented, chip returns $[x|0]_{PrK}$
  - if *auth* PIN was presented, chip returns $[x|1]_{PrK}$
  - if no valid PIN was presented, chip returns error

Electronic signatures, the user interface

- Classical approach: card reader with IC card connected to PC
  - PC has dedicated signing software, e.g., as plugin for a mail client
  - guidance is done on PC screen
  - input must be done done on PC keyboard
- Lots of attack possibilities in the PC
  - intercept PINs, for signing without the card owner
  - show a different message on the screen, etc.
- Attempts at dealing with PC problem
  - tamper-evident, dedicated, non-updateble signature devices
  - like e-book readers, with only a screen, card reader and keypad
  - simplicity and limited functionality allows getting security assurance for such a device
  - not cool: public would prefer a *secure* app on their smartphone
    - this is what IRMA will offer, see privacybydesign.foundation

Example of modern card reader with pin pad

- For use with German e-Identity card *neue Personalausweis (nPA)*
- Interfaces for both contact and contactless cards
- Certified by BSI; cost: 30-50 €

Server-side signatures (beware of snake-oil)

- So far we have assumed that the signer has his private keys locally
  - solid: he signs with ID chip card in dedicated card reader
  - less solid: he signs with his smartphone or laptop
    - concerns: leakage of key pair or loss of private key
- Server-side signature approach:
  - private key is (in secure hardware module) on some remote server
  - keys are very well protected against leakage and loss
  - signer authenticates to server, and then pushes *sign* button
  - different attempt to address non-repudiation
- Problems of server-side signatures
  - can the **sysadmin** sign on behalf of everyone else?
  - strong user authentication requires secret keys anyway
  - example: *Digidentity*
    - uses one-time-password via SMS as user authentication
    - recognized as qualified signatures (what a wonderful world!)

$(\mathbb{Z}_p^*, \times)$ with prime p

Multiplicative prime groups

$(\mathbb{Z}_p^*, \times)$ is a cyclic group of order $p-1$

Alternative way of seeing it:

- Find a generator $g \in (\mathbb{Z}_p^*, \times)$
- Write elements as powers of the generator: g^i
- Multiplication: find c such that $g^c = g^a \times g^b$
- Clearly: $g^a \times g^b = g^{a+b} = g^{a+b \bmod p-1}$
- So $c = a + b \bmod p-1$

$(\mathbb{Z}_p^*, \times)$ is just $(\mathbb{Z}_{p-1}, +)$ in disguise!

Example: $(\mathbb{Z}_{23}^*, \times)$ and $(\mathbb{Z}_{22}, +)$ are similar

Discrete logarithm in $(\mathbb{Z}_p^*, \times)$ with prime p

So: $(\mathbb{Z}_p^*, \times)$ is just $(\mathbb{Z}_{p-1}, +)$ in disguise!

- Let g be a generator of $(\mathbb{Z}_p^*, \times)$
- Let $A = g^a$ and $B = g^b$
  - then $A \times B = g^a \times g^b = g^{a+b \bmod p-1}$
  - multiplication $A \times B$ reduces to addition $a + b$
  - exponentiation A^e reduces to multiplication $a \cdot e$
- Requires knowledge of exponent a (and b), given A (and B)
- Finding this exponent is called discrete log
- Discrete log is hard if p is large

Example:

- discrete exp: find X that satisfies $X \equiv 2^{95} \pmod{149}$
- discrete log: find x that satisfies $2^x \equiv 124 \pmod{149}$

Discrete logarithm problem

Discrete log problem in a cyclic group $\langle g \rangle$

Given $h \in \langle g \rangle$, finding $n < \#g$ that satisfies $h = g^n$

- The discrete log problem is hard in $(\mathbb{Z}_p^*, \times)$ for large p
  - solving a discrete log problem modulo p with p an n-bit prime is about as hard as factoring an n-bit RSA modulus
- It is also hard for many other groups, e.g.,
  - in cyclic subgroups of large order q of $(\mathbb{Z}_p^*, \times)$ with $q \lll p$
  - elliptic curve groups
- Elliptic curve cryptography (ECC) (see later)
  - discrete log in ECC is much harder than for $(\mathbb{Z}_p^*, \times)$
  - for same security strength, compared to RSA:
    - shorter keys, signatures and cryptograms
    - faster key establishment, signing and key pair generation
    - but slower signature verification

Discrete log based crypto: key pairs

- Key pairs:
  - private key: $a \in \mathbb{Z}_{\#g}$
  - public key: $A = g^a \in \langle g \rangle$
  - domain parameters: $\langle g \rangle$, the cyclic group we work in
- Similarities with RSA
  - computing private key from public key is hard problem
  - public key authentication is crucial for security
  - there is mathematical structure
- Differences with RSA
  - domain parameters: you don't have that in RSA
  - key pair generation: take random a and compute $A = g^a$
- Key pairs for $(\mathbb{Z}_p^*, \times)$
  - private key: $a \in \mathbb{Z}_{p-1}$
  - public key: $A = g^a \in \mathbb{Z}_p^*$
  - domain parameters: p and g

Ralph Merkle, Martin Hellman, Whitfield Diffie

Invented public key cryptography in 1976!

(Merkle)-Diffie-Hellman key exchange

- public-key based establishment of a shared secret
- Alice and Bob establish a secret key K_{AB}
  - Alice has $PrK_{\text{Alice}} = a$ and $PK_{\text{Alice}} = A \ (= g^a)$
  - Bob has $PrK_{\text{Bob}} = b$ and $PK_{\text{Bob}} = B \ (= g^b)$
- The protocol (simple static flavour): exchange of public keys

$$\text{Alice} \longrightarrow \text{Bob}: A$$
$$\text{Bob} \longrightarrow \text{Alice}: B$$

- Computation of the shared secret:
  - Bob uses his private key b to compute $K_{AB} = A^b$
  - Alice uses her private key a to compute $K_{AB} = B^a$
  - Correctness: $A^b = (g^a)^b = g^{a \cdot b} = (g^b)^a = B^a$

Diffie-Hellman key exchange: attention points

- Security
  - eavesdropper Eve needs either a or b to compute K_{AB}
  - given $\langle g \rangle$, A and B, predicting K_{AB} should be hard
  - called the (decisional) Diffie-Hellman hardness assumption
  - seems as hard as the discrete log problem but no proof (yet)
- Domain parameters: both need to work in the same cyclic group
- Public key authentication
  - If Alice validated Bob's public key, she knows only Bob has K_{AB}
  - If Bob validated Alice's public key, he knows only Alice has K_{AB}
- Entity authentication?
  - can be done with symmetric crypto challenge-response using K_{AB}
  - along with encryption, message authentication
  - often one uses $h(K_{AB})$ for deriving symmetric keys from K_{AB}

Diffie-Hellman key exchange: cases

- Mutual authentication: both parties authenticate public keys
- Unilateral authentication:
  - Alice authenticates Bob's public key but not vice versa
  - Alice still has guarantee that Bob is only other party having K_{AB}
  - only Bob can decipher what she enciphers with K_{AB}
  - only Bob can generate MACs with K_{AB}
- TLS (https) mostly uses unilateral authentication
  - browser authenticates public key of website
  - website does not authentication public key of browser
- Static Diffie-Hellman: Alice and Bob have long-term keys
  - limitation: K_{AB} is always the same
  - for symmetric crypto: requires nonces across multiple sessions
  - leakage of K_{AB}, a or b allows decryption of all past cryptograms
  - wish for forward secrecy: leakage of K_{AB}, a or b not affecting past cryptograms

Diffie-Hellman key exchange with forward secrecy

- Consider unilateral case where Bob does not validate Alice's key
  - Alice can generate fresh keypair (a, A) for each session/message
  - this is called an ephemeral key pair
  - leaking K_{AB} or a only affects single session/message
  - leaking b still affects all communication between Alice and Bob
- Dynamic Diffie-Hellman
  - Alice generates ephemeral key pair (a, A) per session
  - Bob generates ephemeral key pair (b, B) per session
  - auth. of A: Alice signs (Alice, A, N) with long term PrK_A
  - Bob verifies Alice's signature using the validated PK_A
  - in mutual authentication: also vice versa
  - now leakage of K_{AB}, a or b only affects a single session
  - after the session Alice deletes K_{AB} and a, Bob deletes K_{AB} and b
  - this offers forward secrecy
- Ephemeral key pairs in RSA would work too but very expensive

Diffie-Hellman in action: e-passports

- We saw the Basic Access Control (BAC) protocol for e-passports
  - terminal access to passport chip via Machine Readable Zone (MRZ)
  - restricted to less sensitive data, also on the passport paper
- There is also an Extended Access Control (EAC) protocol
  - for the more sensitive biometric date, like fingerprints (EAC is done after BAC)
  - introduced later (since 2006) by German BSI
  - involves two subprotocols
    - Chip Authentication (CA), with certified public key from chip, ephemeral key pair from terminal
    - Terminal Authentication (TA), with certified key pair from terminal: for giving access to biometric data
  - Here we sketch how CA works

Chip Authentication (from EAC)

$$PsP \xrightarrow[(s_p \text{ is static DH key})]{[g^{s_p}, Id]_{PrK_{\mathbf{CA}}}} Rdr$$

$$PsP \xleftarrow[(s_R \text{ is ephemeral DH key})]{g^{s_R}} Rdr$$

$K = g^{s_p s_R}$:fresh shared secret;
derived to two keys: $K_{\text{enc}}, K_{\text{mac}}$

$$PsP \xrightarrow{K_{\text{mac}}\{g^{s_R}\}} Rdr$$

Rdr now authenticated PsP as it knows

- PsP must have shared secret K
- so PsP has private key s_P matching the public key g^{s_P}

NSA breaking encrypted connections [for info]

CCS 2015 paper *Imperfect Forward Secrecy: How Diffie-Hellman Fails in Practice* explains:

- Diffie-Hellman is used for VPNs, https websites, email, etc.
- Many implementation use the same domain parameters
  - a 1024 bit prime p
  - a particular generator $g \in \mathbb{Z}_p$
- A very large look-up table can be compiled
  - to efficiently solve discrete log in this group
  - authors estimate that this could be done for $100M
  - NSA may have budget for that
- This could explain suggestions in Snowden documents that the NSA has access to encrypted connections.

Student feedback after exam in 2012 ☺

El Gamal: discrete-log based encryption

Encryption with public key A

- convert cleartext M to element $m \in \langle g \rangle$
- randomly generate ephemeral key pair $(r, R = g^r)$
- define cryptogram as $\{m\}_A = (R,\ m \cdot A^r)$
- multiplying m with random A^r and giving R as side info

Decryption with private key a — such that $A = g^a$

- Assume ciphertext $c = (c_1, c_2)$, with $c_i \in \langle g \rangle$
- define recovered plaintext as $[(c_1, c_2)]_a = \dfrac{c_2}{(c_1)^a}$
- removing the factor A^r by dividing by $R^a = g^{ar} = A^r$

Correctness

- For $A = g^a$ we get:

$$[\{m\}_A]_a = [R, m \cdot (g^a)^r]_a = \frac{m \cdot g^{a \cdot r}}{(g^r)^a} = \frac{m \cdot g^{a \cdot r}}{g^{a \cdot r}} = m$$

DSA: discrete-log based signatures

Signing with private key a of message m

- randomly generate ephemeral key pair $(r, R = g^r)$ with $\gcd(r, \#g) = 1$

$$\mathrm{sign}_a(m) = \left(R,\ \frac{h(m) - a \cdot R}{r} \bmod \#g\right)$$

Verification of $m, (s_1, s_2)$ with public key $A \in \langle g \rangle$

- check the equation:

$$g^{h(m)} \stackrel{??}{=} (s_1)^{s_2} \cdot A^{s_1}$$

Notice: no decryption, just checking

Correctness

- $r \cdot s_2 \equiv h(m) - a \cdot R = h(m) - a \cdot s_1 \bmod \#g$ so that:
- $h(m) \equiv r \cdot s_2 + a \cdot s_1 \pmod{\#g}$ and so:
- $g^{h(m)} = g^{r \cdot s_2 + a \cdot s_1} = (g^r)^{s_2} \cdot (g^a)^{s_1} = R^{s_2} \cdot (g^a)^{s_1} = (s_1)^{s_2} \cdot A^{s_1}$

Example calculation for El Gamal

Take $G = \mathbb{Z}_p$ for $p = 107$ and $g = 10 \in G$ with order $q = 53$.

- **Keys:** private $a = 16$; public $A = g^a = 10^{16} = 69 \bmod 107$
- **Encryption:** of $m = 100 \in G$ with random $r = 42$ gives:

$$\{m\}_A = (g^r, A^r \cdot m) = (10^{42}, 69^{42} \cdot 100) = (4, 11)$$

- **Decryption:** of $(4, 11)$ is $\frac{11}{4^a}$
  - $4^a = 4^{16} = 29$ and $\frac{1}{29} = 48 \bmod 107$
  - Hence $\frac{11}{4^a} = 11 \cdot 48 = 100 \bmod 107$

(For modular calculation use eg: `http://ptrow.com/perl/calculator.pl`)

Example calculation for DSA

Still with the same $p = 107, g = 10, q = 53, a = 16, A = 69$,

- **Sign:** $H(m) = 100$ with random $r = 33$
  - We have $g^r = 10^{33} = 102 \bmod 107$
  - and: $\frac{1}{r} = \frac{1}{33} = 45 \bmod 53$
  - next:

$$\frac{H(m) - a \cdot g^r}{r} = (100 - 16 \cdot 102) \cdot 45 = 5 \cdot 45 = 13 \bmod 53$$

  - The signature is thus: $(102, 13)$.
- **Verification:** of $(s_1, s_2) = (102, 13)$
  - first, $g^{H(m)} = 10^{100} = 34 \bmod 107$
  - and also: $(s_1)^{s_2} \cdot A^{s_1} = 102^{13} \cdot 69^{102} = 62 \cdot 4 = 34 \bmod 107$.

Background [for info]

- The primes $p = 107$ and $q = 53$ in the example satisfy $p = 2q + 1$
- The order of $\mathbb{Z}_p^*$ is $p - 1 = 2q$
- $\langle g \rangle$ is a subgroup of $\mathbb{Z}_p^*$ with order q
  - all elements in $\langle g \rangle$, except 1, have order q
  - If this subgroup is of prime order q, then the "Decisional Diffie-Helmann" assumption is believed to hold
- We say we have an embedding of groups:
  - group $(\mathbb{Z}_p^*, \times)$ of order $p - 1$ with p a prime
  - group $\langle g \rangle$ of order $\#g = q$ with q a prime, $(\mathbb{Z}_q, +)$ in disguise
- For some security strength s, q can be taken much smaller than p
  - e.g. for $s = 128$, 256-bit $|q| \geq 256$ is enough but $|p| \geq 3000$
  - decreasing $|p|$ or $|q|$ would reduce security s
  - see `www.keylength.com`

Background on Elliptic Curve Cryptography (ECC)

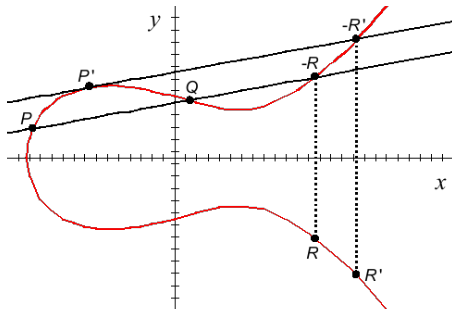
- Koblitz and Miller proposed the use of elliptic curves for cryptography in the mid 1980's
  - group operation is given by addition of points on a curve
  - mainstream public key crypto nowadays: TLS 1.3, e-passports, ...
- Allows discrete log based crypto in EC groups
  - EC Diffie-Hellman, EC Elgamal, EC DSA
  - but much shorter public keys for same security strength s
  - richer functionality, e.g., pairings (advanced, cool topic)
- Key lengths (in bits) for comparable strength (source: NIST):

security strength	modulus length RSA and classical RSA	ECC
80	1024	160
128	3072	256
256	15360	512

Addition on an elliptic curve over the real numbers

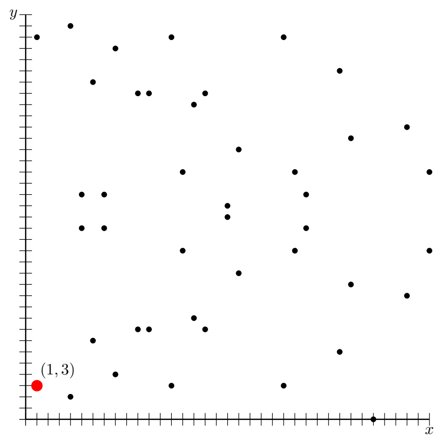
Elliptic curves are given by equations such as: $y^2 = x^3 + ax + b$

Addition $P + Q = R$ and $P' + P' = 2 \cdot P' = R'$ is given by:

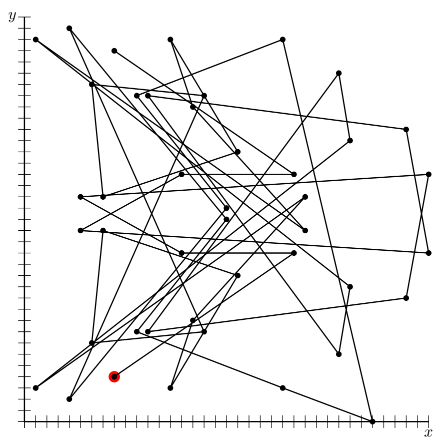


There are also explicit formulas for such additions.

Example curve: $y^2 = x^3 + 2x + 6$ over finite field $\mathbb{Z}_{37}$



Repeated addition: $n \cdot P$ goes everywhere



Given $Q = n \cdot G$, finding n involves basically trying all options

Discrete Log and public keys for ECC

ECC uses additive notation so discrete log problem looks a bit funny:

scalar multiplication: $[n] \cdot G = G + \cdots + G$
Given $[n] \cdot G$ and G, it is hard to find the scalar n.

Key pairs in ECC:

- Domain parameters: the prime p, the constants a and b, generator G and its order $\#G$
- Private key: an integer $a \in \mathbb{Z}_{\#G}$
- Public key: a point on the curve $A = [n]G$

On PGP (by Phil Zimmermann, 1991)

Use fresh session key K for efficiency:

$$A \longrightarrow B\colon \{K\}_{PK_B},\ K\{m,\ [h(m)]_{PrK_A}\}$$

This is basically what PGP (= Pretty Good Privacy) does, e.g., for securing email. It is efficient, because m may be large.

Needham-Schroeder two-way authentication

- Simple protocol, originally proposed in 1978
- uses RSA encryption to achieve authentication
- Serious flaw discovered only in 1996 by Gavin Lowe
  - required formal methods, namely model checking
- Can simply be fixed
- Fix can be seen as just applying appropriate domain separation

Needham-Schroeder: original version + attack

Protocol

$$\begin{aligned} A &\longrightarrow B\colon \{A, N_A\}_{PK_B} \\ B &\longrightarrow A\colon \{N_A, N_B\}_{PK_A} \\ A &\longrightarrow B\colon \{N_B\}_{PK_B} \end{aligned}$$

Attack

$$\begin{aligned} A &\longrightarrow T\colon \{A, N_A\}_{PK_T} \\ T &\longrightarrow B\colon \{A, N_A\}_{PK_B} \\ B &\longrightarrow T\colon \{N_A, N_B\}_{PK_A} \\ T &\longrightarrow A\colon \{N_A, N_B\}_{PK_A} \\ A &\longrightarrow T\colon \{N_B\}_{PK_T} \\ T &\longrightarrow B\colon \{N_B\}_{PK_B} \end{aligned}$$

Interpretation of the attack

If A is so silly to start an authentication with an untrusted T (who can intercept), this T can make someone else, namely B, think he is talking to A while he is talking to T.

Needham-Schroeder: a fix

$$\begin{aligned} A &\longrightarrow B\colon \{A, N_A\}_{e_B} \\ B &\longrightarrow A\colon \{N_A, B, N_B\}_{e_A} \\ A &\longrightarrow B\colon \{N_B\}_{e_B} \end{aligned}$$

Blind signatures: what is the point?

- Suppose A wants B to sign a message m, where B does not know that he signs m
  - Compare: putting an ordinary signature via a carbon paper
- Why would B do such a thing?
  - for anonymous "tickets", e.g., in voting or payment
  - the private key may be related to a specific (timely) purpose
  - hence B does have some control
- Blind signature were introduced in the earlier 80s by David Chaum

Blind signatures with RSA

Let (n, e) be the public key of B, with private key (n, d).

(1) A wants to get a blind signature on m; she generates a random r, computes $m' = (r^e) \cdot m \bmod n$, and gives m' to B.

(2) B signs m', giving the result $k = [m']_{(n,d)} = (m')^d \bmod n$ to A

(3) A computes:

$$\frac{k}{r} = \frac{(m')^d}{r} = \frac{(r^e \cdot m)^d}{r} = \frac{r^{ed} \cdot m^d}{r} \equiv \frac{r \cdot m^d}{r} = m^d = [m]_{(n,d)}$$

Thus: B signed m without seeing it!

Blind signatures for e-voting tickets

- Important requirements in voting are (among others)
  - vote secrecy
  - only eligible voters are allowed to vote (and do so only once)
- There is a clear tension between these two points
- Usually, there are two separate phases:
  (1) checking the identity of voters, and marking them on a list
  (2) anonymous voting
- After step 1, voters get a non-identifying (authentic, signed) ticket, with which they can vote
- Blind signatures can be used for this passage from the first to the second phase

Blind signatures for untraceable e-cash

Assume bank B has key pairs (e_x, d_x) for coins with value x

$C \longleftrightarrow B$: authentication steps
$C \longrightarrow B$: "I wish to withdraw €15, as a €5 and a €10 coin"
$C \longrightarrow B$: $r_1^{e_5} \cdot h(c_1), r_2^{e_{10}} \cdot h(c_2)$ (with r_i, c_i random)
$B \longrightarrow C$: $\left(r_1^{e_5} \cdot h(c_1)\right)^{d_5} = r_1 \cdot h(c_1)^{d_5}, \left(r_2^{e_{10}} \cdot h(c_2)\right)^{d_{10}} = r_2 \cdot h(c_2)^{d_{10}}$

As a result

- C can spend signed coins $(c_1, h(c_1)^{d_5}, 5)$; value is checkable
- the bank cannot recognise these coins: this cash is untraceable
- double spending still has to be prevented (either via a database of spent coins, or via more crypto)

Authorities don't want such untraceable cash, because they are afraid of black markets and losing control (see Bitcoin, later on)

Overview: security goals and public-key crypto

- **Confidentiality**

$$A \longrightarrow B\colon \{m\}_{PK_B}$$

More efficiently, via a (symmetric) session key K:

$$A \longrightarrow B\colon \{K\}_{PK_B}, K\{m\}$$

- **Authentication** Challenge-response with nonce N

$$\begin{array}{l} A \longrightarrow B\colon \{N, A\}_{PK_B} \\ B \longrightarrow A\colon N \end{array} \qquad \text{or} \qquad \begin{array}{l} A \longrightarrow B\colon N \\ B \longrightarrow A\colon [N, B]_{PrK_B} \end{array}$$

- **Integrity** & **non-repudiation**, with hash function h,

$$A \longrightarrow B\colon m, [h(m)]_{PrK_A}$$