Category Theory – Exercise Sheet 9

December 3, 2013

The deadline is 6pm on Thursday the 5th of December. You can either email your answers to `r.furber` at `cs.ru.nl` (do not forget the attachment) or put them in Robert Furber's postbox on the north side of the second floor of the Huygensgebouw. It is inside a white cabinet with its back to the stairs, opposite the photocopier. Please fasten the sheets of paper together firmly with a staple or paperclip. Folding over the corner is not good enough.

For each question, the number of marks available is indicated in round brackets. The total number is 50.

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(a) Let $\mathcal{C}, \mathcal{D}$ be categories, and $X, Y \in \mathrm{Obj}(\mathcal{C})$. Show that if $F : \mathcal{C} \to \mathcal{D}$ is fully faithful, and $F(X) \cong F(Y)$, then $X \cong Y$. (6pt)

Now let $\mathcal{C}$ be a bicartesian closed category.

(b) Show that the exponential A^0 is terminal for all objects $A \in \mathrm{Obj}(\mathcal{C})$, where 0 is the initial object. (6pt)

(c) Show, using the Yoneda lemma, that in any bicartesian closed category $\mathcal{C}$ (11pt)

$$C^{A+B} \cong C^A \times C^B$$

for all objects $A, B, C \in \mathrm{Obj}(\mathcal{C})$. You are not required to prove naturality of all the isomorphisms involved.

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Let $F, G \in \mathbf{Sets}^{\mathcal{C}^{\mathrm{op}}}$ be two presheaves, with two parallel maps $\sigma, \tau \colon F \Rightarrow G$ between them. For each $X \in \mathcal{C}$, let $G(X) \twoheadrightarrow H(X)$ be the coequalizer of $\sigma_X, \tau_X \colon F(X) \to G(X)$ in **Sets**. Show

(a) that the mapping $X \mapsto H(X)$ extends to a functor $\mathcal{C}^{\mathrm{op}} \to \mathbf{Sets}$ such that the maps $G(X) \to H(X)$ form a natural transformation (14pt)

(b) that is the coequalizer of σ, τ in $\mathbf{Sets}^{\mathcal{C}^{\mathrm{op}}}$. (13pt)