Category Theory – Exercise Sheet 7

October 23, 2013

The deadline is 6pm on Thursday the 14th of November. You can either email your answers to `r.furber` at `cs.ru.nl` (do not forget the attachment) or put them in Robert Furber's postbox on the north side of the second floor of the Huygensgebouw. It is inside a white cabinet with its back to the stairs, opposite the photocopier. Please fasten the sheets of paper together firmly with a staple or paperclip. Folding over the corner is not good enough.

For each question, the number of marks available is indicated in round brackets. The total number is 50.

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(a) Show that the family of maps $\mathrm{eval} : A^C \times C \to A$ defines a natural transformation $\mathrm{eval} : -^C \times C \Rightarrow \mathrm{Id}$. (5pt)

(b) Show that $(A \times B)^C \cong A^C \times B^C$ where $A, B, C \in \mathrm{Obj}(\mathbb{C})$ and $\mathbb{C}$ is a cartesian closed category. (15pt)

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Show that in any bicartesian closed category $\mathbb{C}$, for each $C \in \mathrm{Obj}(\mathbb{C})$ we have that the functor $- \times C : \mathbb{C} \to \mathbb{C}$ preserves binary coproducts and the initial object (hence finite coproducts). (20pt)

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Consider the "functor" category $\mathbb{D}^{\mathbb{C}}$, which has functors $F : \mathbb{C} \to \mathbb{D}$ as objects, and for which the maps $F \to G$ are natural transformations $\alpha : F \Rightarrow G$. Composition is given by $(\alpha \circ \beta)_X = \alpha_X \circ \beta_X$ and the identity arrows are $\mathrm{id}_F : F \to F$, having components $(\mathrm{id}_F)_X = \mathrm{id}_{F(X)}$. You are not required to prove that this makes a category.

Show that $\alpha : F \Rightarrow G$ is an isomorphism in $\mathbb{D}^{\mathbb{C}}$ iff α_X is an isomorphism for all $X \in \mathrm{Obj}(\mathbb{C})$. (10pt)