

# Category Theory – Exercise Sheet 3

September 27, 2013

The deadline is 6pm on Thursday the 3rd of October. You can either email your answers to `r.furber` at `cs.ru.nl` (do not forget the attachment) or put them in Robert Furber's postbox on the north side of the second floor of the Huygensgebouw. It is inside a white cabinet with its back to the stairs, opposite the photocopier. Please fasten the sheets of paper together with a staple or paperclip.

For each question, the number of marks available is indicated in round brackets. The total number is 50.

## 9

Let  $\mathbf{C}$  be a category with products and terminal object  $1$ . Take objects  $A, B, C \in \mathbf{C}$  and  $f, g \in \text{Arr}(\mathbf{C})$  arrows in  $\mathbf{C}$ . Prove the following:

(a)  $A \times B \cong B \times A$  (6pt)

(b)  $1 \times A \cong A \cong A \times 1$  (9pt)

(c) If  $f$  and  $g$  are monic, so is  $f \times g$ . (6pt)

(d) If  $f$  is monic, so is  $\langle f, g \rangle$ . (4pt)

## 10

Recall **JSL**, the category of join-semilattices from exercise 6 last week. (17pt)  
Show, giving all details, that **JSL** has finite products.

## 11

The category **Rel** has sets as objects and maps  $X \rightarrow Y$  are relations  $R \subseteq X \times Y$ . Show, giving all details, that **Rel** has finite coproducts. Show that these are also products. (8pt)